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THE OPTIMIZATION OF CONTINUOUS DISCOUNT FUNCTION THROUGH THE CONTINUOUS YIELD CURVE TRAJECTORIES INVOLVING THE PARSIMONIOUS NELSON-SIEGEL MODEL

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ABSTRACT

Yield curves which explain the variation in interest rates over varying maturities, provide important indicators with potentially useful implications on the investment choices, risk management strategies of financial institutions and fiscal policies of regulators. Accurately estimating the parsimonious parameters of the yield curve and modelling the discount function is highly important. The continuous forward rate function is a transformed discount function as a numerical device employed to obtain the price of a collection of bonds since the present value of a set of cash flows is computed by finding the product of these cash flows and the associated discount function. Nigeria's bond market representing a major segment of the capital market and an important means of monetary transmission framework has suffered continuous liquidity problems. The lack of functional capital market instruments in form of government bonds has accounted for the poor performance of bond market. Consequently, this study intends to (i) Compute the discount function and (ii) Show its applications on life insurance offerings. This study analysed the term structure of interest rates using the Nigerian Eurobond which was collected for the years 2018 and 2019 from which the discount functions were constructed by adopting the continuous Nelson Siegel continuous function. A t-test conducted on the adjusted R^2 on the model using Ordinary Least Square method after fixing the shape parameters was applied after computing the exponentially decaying factor. Computational evidence from the results reveals that the adjusted Rsquare for 2018 *and* 2019 *are* 0.976*and respectively*0.986*. This indicates that* 97.6%*of* 2018 *and* 98.6% *of the observed data can be explained by the Nelson-Siegel model of data using the estimated parameters for the two years and hence the result suggests a high level of accuracy confirming the degree to which the Nelson-Siegel yield curve parameters has estimated the illiquid Nigerian financial market.*

Keywords: Discount function, Yield Curve, Term Structure, Eurobond, Nelson-Siegel Model

1. Introduction

The term structure of interest rates is defined as function of: the zero coupon rates, the discount rates and the forward rates such that the forward function trajectories determines rates as a function of the time to maturities. The zero coupon rate is a rate which pays no interest but generates return at maturity. The yield curve is a curve that plots yields or interest rates of the bond which have equal credit quality but different maturity dates. The forward rates defines an interest rate applicable to a financial transaction that will take place in the future. It is estimated from the spot rate and are adjusted to determine the future interest rates. The discount rate is the interest rate that is applied to future cash flows of an investment in order to compute its present value. The yield curve estimation offers critical explanation of debt instruments portfolio construction & market risk management and interest rate density derivations. Because the discount function could be obtained from the zero coupon function, it is apparently the analytical transformations of the continuous forward rate function. Following (Filipovic, 2009, Chakroun, & Abid, 2014; Walli, & Bari, 2018), the discount function is a mathematical device used to obtain the price of a collection of bonds since the present value of a set of cash flows is computed by finding the product of these cash flows and the associated discount function.

Therefore, the discount function defines the underlying element of the model associating the continuous forward rate function with bond prices. Consequently, the changing level of the parsimonious parameter values in the discount rate function that are correspondingly different in the zero coupon and the continuous forward rate function is associated with varying degree of the discount factors and hence changing bond prices is obtained. In (Filipovic, 2009, Chakroun, & Abid, 2014; Walli, & Bari, 2018; Castello and Resta, 2019), the term structure of interest rates trajectory is described by the yield curves motion that have been potentially derived from large number of prices associated with debt instruments. The average yield curve representing the average of the parsimonious parameters of the exponential terms is assumed to be increasing and concave. Nigeria is embarking on some key economic reforms and therefore being exposed to an avalanche of financial uncertainties.

It is therefore under the surveillance of the institutional investors consequently, the examination of its yield curve is important to the regulators of financial institutions and institutional investors to measure financial risks connected with market instruments. Unfortunately, from all the references cited above such as (Filipovic, 2009, Chakroun, & Abid, 2014; Walli, & Bari, 2018; Castello and Resta, 2019), it is observed that little or no efforts in the form of literature have been contributed to key problem areas of Nigerian yield curve forecasting but rather they concentrate on the yield curves forecasting of the advanced economies. Nigeria’s bond market representing a major component of the capital market and an important means of monetary transmission mechanism has suffered continuous liquidity problems. The inadequacy of debt instruments have been responsible for the poor performance of the capital market operations. In its capital market, there are many kinds of debt instruments of which zero coupon bond is one and the interest rate earned on this bond is the yield. The yield represents the market expectation and at same time relies on the interest rate trajectories based on the market price at a definite time. Given market operator’s risk aversion, the demand for the lowest risky instrument is bigger than other risky instruments. The corporate bonds issued by firms seem riskier than government bonds and this accounts for the reason why the risk premium is embedded in bond prices relative to government bonds.

Consequently, the risk premium reveals a higher interest rates at every maturity date which describes a yield curve identical to government bonds but shifted upwards. The risk conceived by market operators on the bond from different issuing economies changes in accordance with the quality of their signatures. In practice, the Nelson-Siegel model is often applied in the analysis and hedging of the interest rate risk of insurance portfolios with defined flows. It has been successfully adopted by many central banks and fixed income portfolio managers. For example (Klaus, Marliese, and Marc, 2000) applied the model to derive a zero coupon rates modified to the German bond market.

In (Svensson, 1994; Saunders & Cornett, 2014; Javier, 2009; Stelios & Avdoulas, 2020; Stuart, 2020), the forward rate function defines the interest of a forward contract on an investment which is initiated $\overline{\xi}$ into the future and maturing $\underline{\xi}$ periods beyond the commencement date of the contract. The continuously instantaneous forward rate $g(\xi)$is obtained by tending the maturity of the forward contract to zero and consequently, $\lim_{\underline{\xi}\to 0} g\left(\underline{\xi},\overline{\xi}\right) = g(\overline{\xi})$.

so that the forward rate $g(\xi)$can be obtained from the instantaneous forward rates. Furthermore, the spot rate $g(\overline{\xi})$implicit in the discount bond with maturity $\overline{\xi}$ can be obtained. In order to derive the yield function $\Upsilon(\xi)$, we apply the mean value theorem for integrals as follows

$$\Upsilon(\overline{\xi}) = \frac{1}{\xi}\int_0^{\xi} g(\theta)d\theta \tag{1}$$

From equation (1), the discount function is then obtained as follows

$$D(\xi) = e^{-\int_0^{\xi} g(\theta)d\theta} \tag{2}$$

Taking the logarithms of both sides of (2), we obtain

$$log_e D\,(\xi) = -\int_0^{\xi} g(\theta)d\theta \tag{3}$$

Differentiating both sides of (3), we obtain

$$\frac{D'(\xi)}{D(\xi)} = -g(\theta) \tag{4}$$

2. Material and Methods

2.1 Nelson-Siegel Class

The Nelson-Siegel originally assumes that the forward rates follow the expression written in matrix form as:

$$(\alpha_0 \quad \alpha_1 \quad \alpha_2) \times \begin{pmatrix} 1 \\ e^{-\lambda_t \tau} \\ \tau\lambda e^{-\lambda_t \tau} \end{pmatrix} = (\alpha_o \quad \alpha_1 \quad \alpha_2) \times \begin{pmatrix} g_0 \\ g_1 \\ g_2 \end{pmatrix} \tag{5}$$

The model consist of a constantg_0; an exponential decay function g_1 and a Laguerre function$g_2$derived from the second order differential equation. The Laguerre function is the product of an exponential with a polynomial. Mathematically, the forward rate function can then be expressed as: $\alpha_0 g_0 \ + \alpha_1 g_1 \ + \alpha_2 g_2$ implies that the expression is in the form

$$g_t(\tau) = \alpha_{0t} + \alpha_{1t} e^{-\lambda_t \tau} + \alpha_{2t} \tau \lambda e^{-\lambda_t \tau} \tag{6}$$

where$\alpha_{0t}, \alpha_{1t}, \alpha_{2t}$and λ_t are time varying parameters respectively called the level, slope and curvature factors. $g_t(\tau)$ is the zero-coupon rate at observed time t with maturity time τ. Following (Diebold and Li, 2006), the above Nelson-Siegel parameters were reformulated to fall in line with meaningful financial interpretations.

The yield curve (yield as a function of maturity) can also be obtained as below.

$$\Upsilon_t(\tau) = \frac{1}{\tau} \int_0^\tau g_t(u) du \tag{7}$$

This implies that:

$$\Upsilon_t(t) = \frac{1}{\tau} \int_0^\tau \alpha_{0t} + \alpha_{1t} e^{-\lambda_t u} + \alpha_{2t} \lambda e^{-\lambda_t u} du \tag{8}$$

$$\Upsilon_t(\tau) = \frac{1}{\tau} \left(\alpha_{0t} \tau + \alpha_{1t} \frac{1 - e^{-\lambda_t \tau}}{\lambda_t} + \alpha_{2t} \int_0^\tau \lambda_t u e^{-\lambda_t u} du \right) \tag{9}$$

$$\Upsilon_t(\tau) = \alpha_{0t} + \alpha_{1t} \frac{1 - e^{-\lambda_t \tau}}{\tau \lambda_t} + \alpha_{2t} \left(-e^{-\lambda_t \tau} + \frac{1 - e^{-\lambda_t \tau}}{\tau \lambda_t} \right) \tag{10}$$

$$\Upsilon_t(\tau) = \alpha_{0t} + (\alpha_{1t} + \alpha_{2t}) \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - \alpha_{2t} e^{-\lambda_t \tau} \tag{11}$$

λ determines the decay speed of the $\alpha' s$exponential components and the maximum level of the $\alpha' s$exponential component. Consequently, λ controls the decaying rate of the entire curve. In its raw form, the Nelson-Siegel function was developed as a continuous deterministic function. However, (Christensen, Diebold, Rudenbusch, 2007; Christensen, Diebold, Rudenbusch, 2009; Diebold & Rudenbusch, 2013) both obtained a class of arbitrage-free affine dynamic term structure models which estimates the commonly-applied Nelson-Siegel yield-curve specification. The theoretical analysis relates this new class of models to the cannonical representation of the three-factor arbitrage-free affine model. However, the empirical analysis showed that imposing the Nelson-Siegel structure on this canonical representation greatly improves its empirical tractability. Computing the period by period yield using the exponential components of the model, the coefficients $\{\alpha_1, \alpha_2, \alpha_3, \lambda\}$vary with time and hence determine the entire term structure

$\Upsilon_t(\tau_1),\ \ \Upsilon_t(\tau_2), \Upsilon_t(\tau_3),\ \ldots, \Upsilon_t(\tau_m)$ over τin a defined investment period. Although λ_t is assumed to be time varying variable, (Nelson- Siegel, 1987) fixed it so that the model will be linear. The Nelson-Siegel model captures many yield curves shapes to deal with all the shapes that the term structure of interest rates assumes over time especially in an emerging market.

As we take the limiting values of the function $\Upsilon_t(\tau)$ tending to infinity, the resulting value is α_{0t} but if we tend the function to zero, the resulting values is $(\alpha_{0t} + \alpha_{1t})$thus: $\lim_{\tau \to \infty} \Upsilon_t(\tau) = \alpha_{0t}$. Consequently, we can assert that α_{0t} is the long rate. The short rate is the limit of the yield function as τtends to zero

$$\lim_{\tau \to 0} \Upsilon_t(\tau) = \lim_{\tau \to 0} \alpha_{0t} + (\alpha_{0t} + \alpha_{1t}) \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - \alpha_{2t} e^{-\lambda_t \tau} \quad (12)$$

$$\lim_{\tau \to 0} \Upsilon_t(\tau) = \alpha_{0t} - \alpha_{2t} + (\alpha_{1t} + \alpha_{2t}) \lim_{\tau \to 0} \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \quad (13)$$

$$\lim_{\tau \to 0} \Upsilon_t(\tau) = \alpha_{0t} - \alpha_{2t} + (\alpha_{1t} + \alpha_{2t}) \lim_{\tau \to 0} \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \quad (14)$$

$$\lim_{\tau \to 0} \Upsilon_t(\tau) = \alpha_{0t} - \alpha_{2t} + (\alpha_{1t} + \alpha_{2t}) \quad (15)$$

$$\lim_{\tau \to 0} \Upsilon_t(\tau) = \alpha_{0t} + \alpha_{1t} \quad (16)$$

The consol shows a level of higher persistence than the shot rates but depends only on α_{0t}

$$\lim_{\tau \to \infty} \Upsilon_t(\tau) = \lim_{\tau \to \infty} \left[\alpha_{0t} + (\alpha_{0t} + \alpha_{1t}) \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - \alpha_{2t} e^{-\lambda_t \tau} \right] = \alpha_{0t} \quad (17)$$

From equations (16) and (17), it is observed that the difference between the long and short end of the yield curve is the slope. The short rate is reflected and positively dependent on the factor loadings$\alpha_{0t} + \alpha_{1t} > 0$. However, the long rate only depends on α_{0t}. This implies that the short rate of $\Upsilon_t(\tau)$ is $(\alpha_{0t} + \alpha_{1t})$. The limit of the forward rate function $g_t(\tau)$ is the same as of the spot rate. In practice, the short rate is more volatile, more skewed and have higher kurtosis than the consol.

In (Nelson and Siegel, 1987), the shape flexibility was explained differently by interpreting the parameters of the model as measuring the strengths of the short-term, medium-term and long-term components of the forward rate curve. α_{0t} contributes to the long-term component, the contribution of the short-term component is α_{1t} and α_{2t} indicates the contribution of the medium-term component. The medium-term component is the only function within this model that starts out at zero (thus not short-term) and decays to zero (and is therefore not long-term).The short-term component has the fastest decay of both functions within the model that decays monotonically to zero. Observe that the loading $\alpha_{0t}$is a constant which does not depend on the maturity term $\tau$and consequently the term structure at different maturity values will be affected by α_{0t} justifying it being the level factor. The loading of α_{1t} is given by $\frac{1-e^{-\lambda_t\tau}}{\tau\lambda_t}$ and consequently whenever τ approaches zero,

$$\lim_{\tau\to 0}\left(\frac{1-e^{-\lambda_t\tau}}{\tau\lambda_t}\right) = \lim_{\tau\to 0}\left(\frac{\lambda_t e^{-\lambda_t\tau}}{\lambda_t}\right) = 1 \qquad (17a)$$

However, if τ approaches ∞,

$$\lim_{\tau\to\infty}\left(\frac{1-e^{-\lambda_t\tau}}{\tau\lambda_t}\right) = \lim_{\tau\to\infty}\left(\frac{\lambda_t e^{-\lambda_t\tau}}{\lambda_t}\right) = \lim_{\tau\to\infty}\left(e^{-\lambda_t\tau}\right) = 0 \qquad (17b)$$

and the loading converges to zero. As a result, the yield curve is basically affected by α_{1t} in the short term and any change $\Delta\alpha_{1t}$ in the loading translates to a change in the slope of the term structure. Furthermore, the loading of α_{2t} is given by $\left(\frac{1-e^{-\lambda_t\tau}}{\tau\lambda_t} - e^{-\lambda_t\tau}\right)$.

Whenever τ approaches zero, $\lim_{\tau\to 0}\left(\frac{1-e^{-\lambda_t\tau}}{\tau\lambda_t} - e^{-\lambda_t\tau}\right) = \lim_{\tau\to 0}\left(\frac{\lambda_t e^{-\lambda_t\tau}}{\lambda_t} + \lambda_t e^{-\lambda_t\tau}\right) =$

$\lim_{\tau\to 0}\left(e^{-\lambda_t\tau} + \lambda_t e^{-\lambda_t\tau}\right)$

$= 1 + \lambda_t$ (17c)

However, if τ approaches ∞,

$$\lim_{\tau\to\infty}\left(\frac{1-e^{-\lambda_t\tau}}{\tau\lambda_t} - e^{-\lambda_t\tau}\right) = \lim_{\tau\to\infty}\left(\frac{\lambda_t e^{-\lambda_t\tau}}{\lambda_t} + \lambda_t e^{-\lambda_t\tau}\right)$$

$$= \lim_{\tau\to\infty}\left(e^{-\lambda_t\tau} + \lambda_t e^{-\lambda_t\tau}\right) = 0 \qquad (17d)$$

However, from (Diebold and Li, 2006), plotting the loading as a function against time produces the trajectory commencing from point zero which progressively increases and systematically converges to zero again. In the authors' view, the loading factor therefore governs the curvature of the term structure of interest rates and exhibits the most pervasive change effect on the medium-term yield function

2.2 The Application on the Present Value of Life Insurance Future Benefits

Suppose an insured purchases a term insurance with maturity time T. The insured receives death benefits at the end of the year of death when the assured dies between year n and year $n+1$. The death benefit is the higher of the investment at time zero and the fund value at time of death. However, applying the *actuarial assumption to the death benefit*, then $l_x = \int_0^\infty l_{x+t}\mu_{x+t}\,dt$ *lives would die per unit scheme with probability* $F_{T(x)}(t) = 1 - exp\left(-\int_0^t \mu_{x+\xi}d\xi\right)$. The *death benefit at time t* payable upon death will be the greater value of G_t and S_t where

$$G_t = S_0 e^{gt} \qquad (18)$$

Furthermore, the benefit $\boldsymbol{b}_t$ at time t is *assumed* to follow the function defined below

$$\boldsymbol{b}_t = \begin{cases} S_t if S_t > G_t \\ G_t if S_t < G_t \end{cases} \qquad (19)$$

$$\boldsymbol{b}_t = max(G_t, S_t) \qquad (20)$$

$$\boldsymbol{b}_t = max(G_t - S_t, 0) + S_t \qquad (21)$$

$$\boldsymbol{b}_t = (G_t - S_t)^+ + S_t \qquad (22)$$

G_t is the death benefit guaranteed level interest rate g per unit investment over time while S_t is the value of the underlying *debt instrument at time* t representing the accumulation function from time 0 to t where we assume $S_0 = 1$. Life insurance office issues products underwritten with different covenants such as guaranteed rate of interests, bonuses, equity options and unit-linked-contracts forming integral part of the liability of the insurance company. The guaranteed rate of interests has implications on life assurance offerings associated with guaranteed minimum maturity benefits schemes for the assured. The assured pays periodic premium to be invested in debt instruments and thereafter earns benefit at maturity of the policy depending on the performance of the fund. However, there is a guaranteed benefit payable to the policy holder irrespective of the performance of the life fund. The life office is then obliged to pay the guaranteed sum even if the benefit at maturity eventually is smaller than the guaranteed sum. The guarantee which could be regular or equity-dependent increasing provides the downside cover to the scheme holder's fund with the upside cover being participating in the underlying stock index. Following the definitions in Hardy (2003, pp. 100), the present value of the future death benefits is expressed as

$$\boldsymbol{PVFB = b_1 D(0,1) + b_2 D(0,2) + \ldots + b_{T-1} D(0, T-1) + b_T D(0,T)} \quad (23)$$

where $D$is the discount rate and $\boldsymbol{b}$is the benefit

$$\boldsymbol{PVFB = \{(G_1 - S_1)^+ + S_1\}D(0,1) + \{(G_2 - S_2)^+ + S_2\}D(0,2) + \ldots + \{(G_{T-1} - S_{T-1})^+ + S_{T-1}\}D(0,T-1) + \{(G_T - S_T)^+ + S_T\}D(0,T)} \quad (24)$$

2.3 The Guaranteed Minimum Maturity Benefit

Let $\eta \in \boldsymbol{R}$ be a predetermined rate. Suppose the life office issues a T year deferred life annuity payable annually. Then the benefit at time T will be $max(\eta G_T, \eta S_T)$and such that a predetermined rate η is withdrawn or guaranteed annually until death. η is to be numerically estimated as appropriate.

Consequently as defined before, $\boldsymbol{b}_t = \eta(G_t - S_t)^+ + \eta S_t$ (25)

The expectation of $PVFB = \sum_{R=T}^{\infty} \boldsymbol{b}_R D(0,R)$under the risk neutral measure $\boldsymbol{Q}$ becomes

Let $T(x)$ be the complete future life time of a life aged x, then the actuarial present value of benefits is given as follows

$$\boldsymbol{APVFB = \sum_{R=T}^{\infty} E^Q\,[b_R D(0,R)] \times Pr(T(x) > R)} \quad (26)$$

$$\boldsymbol{APVFB = \sum_{R=T}^{\infty} E^Q\,[b_R D(0,R)] \times ({}_Rp_x)} \quad (27)$$

$$\boldsymbol{APVFB = \sum_{R=T}^{\infty} E^Q[b_R D(0,R)] \left(\frac{l_{x+R}}{l_x}\right)} \quad (28)$$

where $\boldsymbol{l_x = \int_0^{\infty} l_{x+t}\mu_{x+t}\,dt}$

(29)

is the number of lives expected to survive at age x

$$\boldsymbol{APVFB = \frac{1}{l_x}\sum_{R=T}^{\infty} E^Q\,[b_R D(0,R)]\,l_{x+R}}$$

(30)

$$\boldsymbol{APVFB} = \frac{\mathbf{1}}{\boldsymbol{l_x}} \sum_{\boldsymbol{R=T}}^{\infty} \boldsymbol{E^Q} \left[\boldsymbol{\eta}\{(\boldsymbol{G_t} - \boldsymbol{S_t})^{\boldsymbol{t}} + \boldsymbol{S_t}\}\boldsymbol{D}(\mathbf{0}, \boldsymbol{R})\right] \boldsymbol{l_{x+R}} \quad (31)$$

$$\boldsymbol{APVFB} = \sum_{\boldsymbol{R=T}}^{\infty} \left\{ \frac{\mathbf{1}}{\left(\int_{\mathbf{0}}^{\infty} \boldsymbol{l_{x+t}\mu_{x+t}dt}\right)} \sum_{\boldsymbol{R=T}}^{\infty} \boldsymbol{E^Q}[\boldsymbol{\eta}\{(\boldsymbol{G_t} - \boldsymbol{S_t})^+ + \boldsymbol{S_t}\}\boldsymbol{D}(\mathbf{0}, \boldsymbol{R})] \times \boldsymbol{l_0} \boldsymbol{e}^{-\int_{\mathbf{0}}^{\boldsymbol{x+R}} \boldsymbol{\mu_t dt}} \right\} \quad (32)$$

where $\mu_x = GM(m, n)$ is the force of mortality

$$\boldsymbol{APVFB} = \left\{ \frac{\mathbf{1}}{\left(\int_{\mathbf{0}}^{\infty} \boldsymbol{l_{x+t}\mu_{x+t}dt}\right)} \sum_{\boldsymbol{R=T}}^{\infty} \boldsymbol{E^Q}\left[\boldsymbol{\eta}\{(\boldsymbol{G_t} - \boldsymbol{S_t})^{\boldsymbol{t}} + \boldsymbol{S_t}\}\boldsymbol{e}^{-\boldsymbol{R} \times \Upsilon(\boldsymbol{R})}\right] \times \boldsymbol{e}^{-\int_{\mathbf{0}}^{\boldsymbol{x+R}} \boldsymbol{\mu_t dt}} \right\} \quad (33)$$

where $\Upsilon_t(\tau) = \alpha_{0t} + (\alpha_{1t} + \alpha_{2t}) \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - \alpha_{2t} e^{-\lambda_t \tau}$ (34)

$$\boldsymbol{APVFB} = \frac{\sum_{\boldsymbol{R=T}}^{\infty} \boldsymbol{E^Q}[\boldsymbol{\eta}\{(\boldsymbol{G_t} - \boldsymbol{S_t})^+ + \boldsymbol{S_t}\}]}{\left(\int_{\mathbf{0}}^{\infty} \boldsymbol{l_{x+t}\mu_{x+t}dt}\right) \boldsymbol{e}^{\boldsymbol{R}\Upsilon(\boldsymbol{R})}} \times \boldsymbol{e}^{-\left(\int_{\boldsymbol{x}}^{\boldsymbol{x+R}} \boldsymbol{\mu_t dt}\right)} \quad (35)$$

Suppose mortality intensity $\mu(x) = BC^x$ is assumed where B and C are the underlying parameters of mortality estimated from A $1967 - 70$ mortality table using algebraic process as follows

$B = 0.000050723$ and $C = 1.10428424$

Then the integrated hazard (total death severity) is

$$\int_x^{x+R} \mu_t dt = \int_x^{x+R} BC^t dt \quad (36)$$

$$\int_x^{x+R} \mu_t dt = \left[\frac{BC^t}{log_e C}\right]_x^{x+R} \quad (37)$$

$$\int_x^{x+R} \mu_t dt = \frac{BC^{x+R}}{log_e C} - \frac{BC^x}{log_e C} \quad (38)$$

$$\int_x^{x+R} \mu_t dt = \frac{BC^x}{log_e C}(C^R - 1) \quad (39)$$

The total severity is obtained as

$$\int_x^{x+R} \mu_t dt = \frac{BC^x}{log_e C}(C^R - 1) \quad (40)$$

$$APVFB = \sum_{R=T}^{\infty} E^{\boldsymbol{Q}} \left[\eta\{(G_t - S_t)^+ + S_t\}e^{-R \times \Upsilon(R)}\right] \times e^{-\left\{\frac{BC^x}{log_e C}\left(e^R - 1\right)\right\}} \quad (41)$$

$$\boldsymbol{APVFB} = \frac{\mathbf{1}}{\boldsymbol{e}^{\boldsymbol{R} \times \boldsymbol{y_t(R)}}} \sum_{\boldsymbol{R=T}}^{\infty} \boldsymbol{E^Q} \left[\boldsymbol{\eta}\{(\boldsymbol{G_t} - \boldsymbol{S_t})^+ + \boldsymbol{S_t}\}\right] \times \boldsymbol{e}^{-\left\{\frac{\boldsymbol{BC^x}}{\boldsymbol{log_e C}}\left(\boldsymbol{C^R} - \mathbf{1}\right)\right\}} \quad (42)$$

Substituting the values of the parameters into (42), we obtain

$$\boldsymbol{APVFB} = \frac{\mathbf{1}}{\boldsymbol{e}^{\boldsymbol{R} \times \boldsymbol{y_t(R)}}} \sum_{\boldsymbol{R=T}}^{\infty} \boldsymbol{E^Q} \left[\boldsymbol{\eta}\{(\boldsymbol{G_t} - \boldsymbol{S_t})^+ + \boldsymbol{S_t}\}\right] \times \boldsymbol{e}^{-\left\{\frac{(\mathbf{0.000050723})(\mathbf{1.10428424})^{\boldsymbol{x}}}{\boldsymbol{log_e}(\mathbf{1.10428424})}\left((\mathbf{1.10428424})^{\boldsymbol{R}} - \mathbf{1}\right)\right\}} \quad (43)$$

The guarantees are important because they pay benefit at maturity and are also associated with the returns of an actively managed investment portfolio.

2.4 Data Analysis

The data presented in this study involves the daily closing of the Nigerian Eurobond yield which comprises of the data from January to December for 2018 and 2019 to analyse and fit the Nigerian Eurobond yield curve using the Nelson-Siegel model. The data for the twelve months were analyzed on quarterly bases for the years 2018 and 2019. The result and findings were also depicted in graphs and tabular form to include other statistics not captured on the curve.

The model can be expressed in a $M \times 3$ matrix in order to estimate the parameters. Given the time to maturity for different bond maturities as $\tau_1, \quad \tau_2 \ ... \tau_m$, and the corresponding yield to maturity as:$\Upsilon_t(\tau_1), \ \Upsilon_t(\tau_2), \Upsilon_t(\tau_3), \ ..., \Upsilon_t(\tau_m)$and based on (Diebold & Rudenbusch, 2007)

we seek to estimate the optimal parameters of the model that is $\alpha_{0t}, \alpha_{1t}, \alpha_{2t}$ and λ_t in terms of best fitting as follows.

$$\begin{bmatrix} 1 & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} - e^{-\lambda_t\tau_1} \\ 1 & \frac{1-e^{-\lambda_t\tau_2}}{\lambda_t\tau_2} & \frac{1-e^{-\lambda_t\tau_2}}{\lambda_t\tau_2} - e^{-\lambda_t\tau_2} \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & \frac{1-e^{-\lambda_t\tau_m}}{\lambda_t\tau_m} & \frac{1-e^{-\lambda_t\tau_m}}{\lambda_t\tau_m} - e^{-\lambda_t\tau_m} \end{bmatrix} \begin{bmatrix} \alpha_{0t} \\ \alpha_{1t} \\ \alpha_{2t} \end{bmatrix} = \begin{bmatrix} \Upsilon_t(\tau_3)(\tau_1) \\ \Upsilon_t(\tau_3)(\tau_2) \\ . \\ . \\ . \\ \Upsilon_t(\tau_3)(\tau_m) \end{bmatrix} \qquad (44)$$

The above matrix is written as:

$$\Upsilon_t(\tau_3)(\tau_j) = \boldsymbol{X}\lambda_t\alpha_t \qquad (45)$$

Where$\Upsilon_t(\tau_3)_t(\tau_j)$ is an M-dimensional vector,$\boldsymbol{X}\lambda_t$ is $M \times 3$and α_t is 3-dimensional vector thus

$$\Upsilon(\tau_3)(\tau_j) = \begin{pmatrix} \Upsilon_t(\tau_1) \\ \Upsilon_t(\tau_2) \\ . \\ . \\ . \\ \Upsilon_t(\tau_m) \end{pmatrix}, \quad \boldsymbol{X}\lambda_t = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} - e^{-\lambda_t\tau_1} \\ 1 & \frac{1-e^{-\lambda_t\tau_2}}{\lambda_t\tau_2} & \frac{1-e^{-\lambda_t\tau_2}}{\lambda_t\tau_2} - e^{-\lambda_t\tau_2} \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & \frac{1-e^{-\lambda_t\tau_m}}{\lambda_t\tau_m} & \frac{1-e^{-\lambda_t\tau_m}}{\lambda_t\tau_m} - e^{-\lambda_t\tau_m} \end{pmatrix}, \quad \alpha_t = \begin{pmatrix} \alpha_{0t} \\ \alpha_{1t} \\ \alpha_{2t} \end{pmatrix}$$

To solve the above system parameters using the matrix $\boldsymbol{X}\lambda_t$ and α_t, Nelson-Siegel suggested that for any $\lambda_t > 0$, the system is computed by applying the ordinary least square technique. Repeating this procedure over a set of values forλ_t gives the overall best-fitting values. Large values of λ_t correspond to rapid decay and therefore will be able to fit

excessive curvature at short maturities well while being unable to fit excessive curvature over longer maturity ranges. Lower values of λ_t leads to slow decay that can fit curvature at longer maturities but they will be unable to follow extreme curvature at short maturities. Essentially, in order to obtain lamda following (Diebold & Canlin, 2006), the third term in equation (10) can be expressed as $\lambda = arg\,\underset{\lambda}{max}\left[\frac{1-e^{-24\lambda_t}}{24\lambda_t} - e^{-24\lambda_t}\right]$
and consequently, using the non-constrained optimization described in (46), we obtained $\lambda = 0.03778438$and such that the loading of the curvature factor achieves its maximum for a maturity of 2 years which is usually observed as the medium term

Table 1. First Quarter Descriptive Statistics 2018

Maturity (τ)	N	Minimum	Maximum	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	44	4.1350	4.8910	4.530432	.0415433	.2755673	.076
4 years	44	4.0140	5.1160	4.827205	.0381422	.2530065	.064
5 years	44	4.8250	6.5880	5.230273	.0590231	.3915147	.153
9 years	44	4.5970	6.6450	6.098409	.0716375	.4751897	.226
14 years	44	5.9210	7.1790	6.711114	.0561927	.3727401	.139
29 years	44	6.8470	7.6730	7.252773	.0412490	.2736152	.075

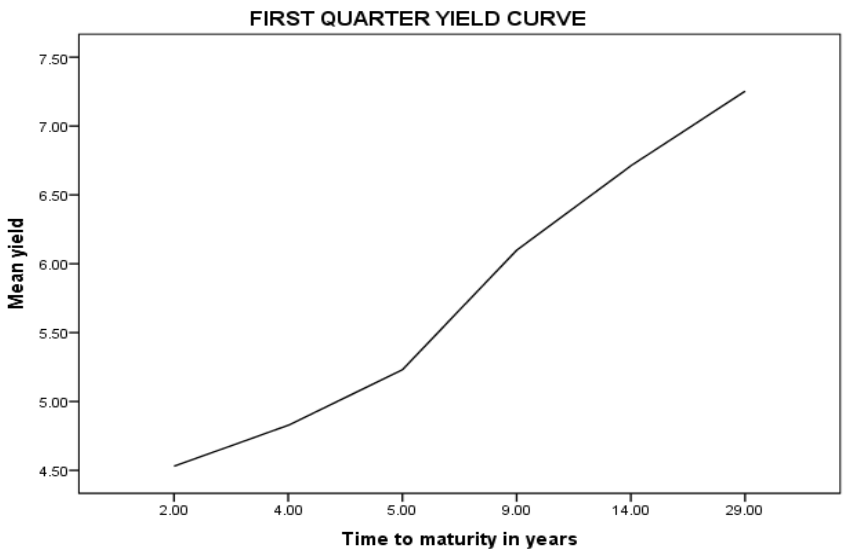

Figure 1: First Quarter Yield Curve

The descriptive analysis of the first quarter 2018 as shown in table 1 above contained six time to maturity of 2 years, 4 years, 5 years, 9 years, 14 years and 29 years and a mean yield of 4.530432, 4.827205, 5.230273, 6.098409, 6.711114 and 7.252773 respectively. The yield curve depicted above on figure1 above shows that the yield curve is moving upward as time to maturity increase.

Table 2. Second Quarter Descriptive Statistics 2018

Maturity (τ)	N	Min	Max	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	45	4.4620	6.2660	5.075422	.0754607	.5062059	.256
4 years	45	4.9100	6.3560	5.352178	.0581200	.3898810	.152
5 years	45	5.0870	6.8500	5.661556	.0700695	.4700406	.221
9 years	45	5.5110	7.6750	6.628289	.0679810	.4560301	.208
12 years	45	6.4940	8.1100	7.089244	.0638245	.4281474	.183
14 years	45	6.6120	8.3260	7.238578	.0663314	.4449644	.198
20 years	45	7.05	8.52	7.5962	.05890	.39511	.156
29 years	45	7.16	8.65	7.6980	.05939	.39842	.159

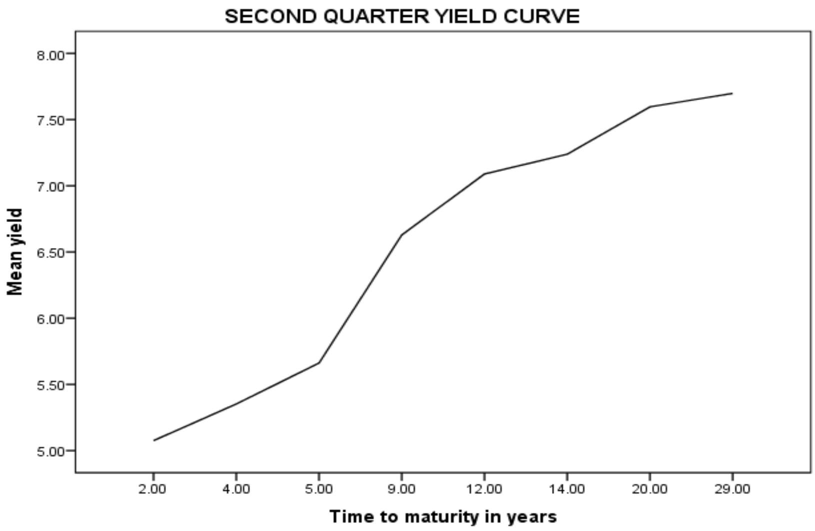

Figure 2: Second Quarter Yield Curve

The number of time to maturity of the Nigerian Eurobond in the second quarter increases with the addition of 12 years maturity and 20 years time to maturity. The time to maturities of the quarter include 2 years, 4years, 5years, 9years, 12years, 14years, 10years and 29years with the corresponding mean yield of 5.075422, 5.352178, 5.661556, 6.628289, 7.238578, 7.5962 and 7.6980.The yield curve shown on figure 2 above reveal that the curve is also moving upward as it is in the previous quarter see table 2

Table 3. Third Quarter Descriptive Statistics 2018

Maturity (τ)	N	Min	Max	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	44	4.6840	5.8460	5.178023	.0456638	.3028993	.092
4 years	44	5.2820	6.1710	5.724455	.0387018	.2567184	.066
5 years	44	5.7710	6.6370	6.190932	.0390991	.2593541	.067
9 years	44	6.7890	7.7790	7.283818	.0432829	.2871063	.082
12 years	44	7.1270	8.1420	7.629477	.0426754	.2830768	.080
14 years	44	7.3980	8.4110	7.867136	.0420926	.2792106	.078
20 years	44	7.71	8.71	8.1622	.04187	.27776	.077
29 years	44	7.83	8.80	8.2826	.03862	.25616	.066

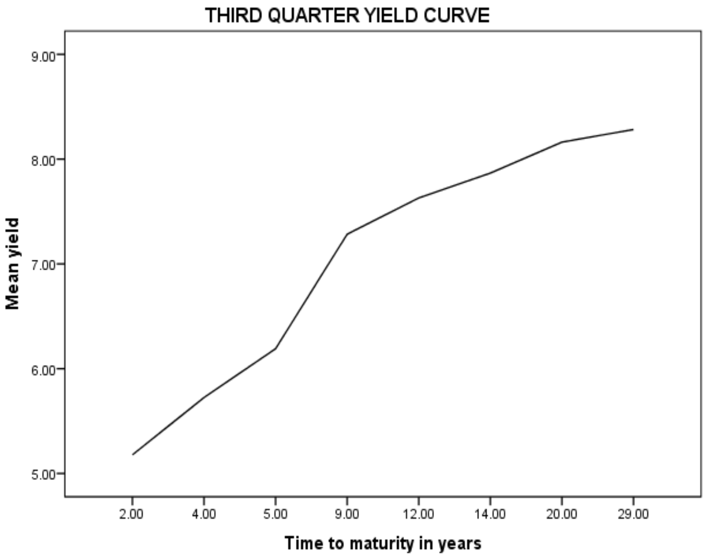

Figure 3: Third Quarter Yield Curve

In the third quarter descriptive statistics as shown above on table 3, the time to maturities available are 2 years, 4years, 5years, 9years, 12years, 14years, 10years and 29years with respective mean yield of 5.178023, 5.724455, 6.190932, 7.283818, 7.629477, 7.867136, 8.16622 and 8.2826. The quarter experienced a higher yield when compared to the previous quarters. The third quarter yield curve as depicted in figure3 above indicated an upward moving slope just as in the previous quarters see table 3.

Table 4. Fourth Quarter Descriptive Statistics 2018

Maturity (τ)	N	Min	Max	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	42	4.8510	6.0730	5.435524	.0661428	.4286541	.184
4 years	42	5.5100	6.5850	6.043381	.0509444	.3301575	.109
5 years	42	5.9790	7.4720	6.687786	.0788105	.5107502	.261
9 years	42	7.0450	8.5820	7.834857	.0702595	.4553336	.207
12 years	42	7.4300	8.8420	8.257643	.0643830	.4172497	.174
14 years	42	7.5480	9.1700	8.469810	.0778415	.5044706	.254
20 years	42	7.90	9.15	8.6805	.05226	.33868	.115
29 years	42	7.99	9.15	8.7056	.04707	.30502	.093

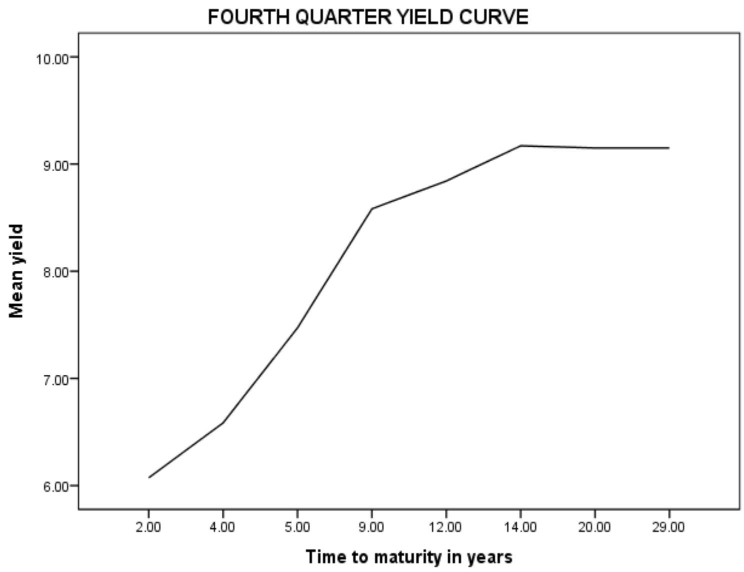

Figure 4: Fourth Quarter Yield Curve

In the fourth quarter descriptive statistics as shown above on table 4, the time to maturities available are 2 years, 4years, 5years, 9years, 12years, 14years, 10years and 29 with respective mean yield of 5.435524, 6.043381, 6.687786, 7.834857, 8.257643, 8.469810, 8.6805 and 8.7056, see table 4. The fourth quarter experiences a tremendous increase in yield. The quarter has a maximum yield of 9.17, 9.15 and 9.15, of 14years, 20years and 29years time to maturity respectively. The yield curve of the fourth quarter as depicted on figure 4 above shows that the yield curve is sloping upward giving an increase of time to maturity.

Table 5. Aggregate Descriptive Statistics 2018

Maturity (τ)	N	Min	Max	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
2 years	175	4.14	6.27	5.0506	.03840	.50792	.258
4 years	175	4.01	6.59	5.4797	.04140	.54767	.300
5 years	175	4.83	7.47	5.9325	.05183	.68565	.470
9 years	175	4.60	8.58	6.9495	.05879	.77778	.605
12years	145	6.49	8.84	7.5734	.05150	.62018	.385
14 years	175	5.92	9.17	7.5595	.05837	.77216	.596
20 years	145	7.05	9.15	8.0615	.04776	.57511	.331
29 years	175	6.85	9.15	7.9749	.04789	.63348	.401

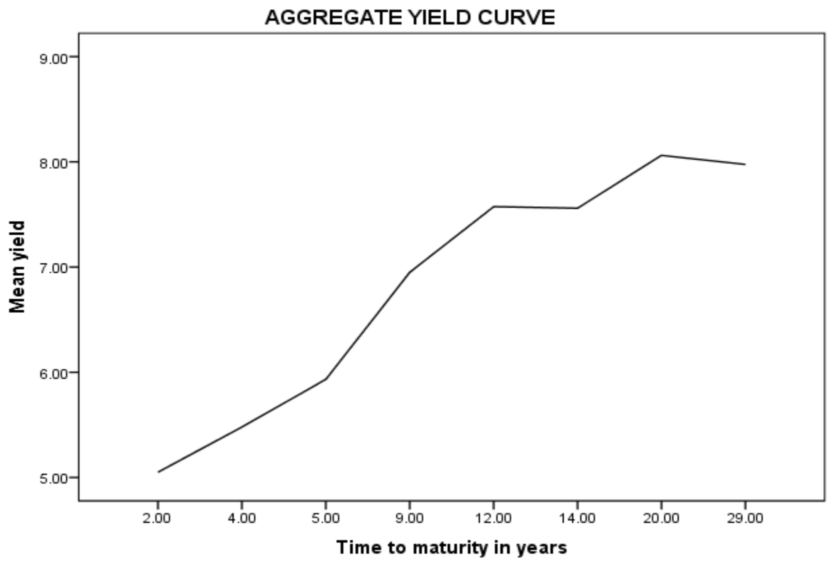

Figure 5: Aggregate Yield Curve

The aggregate descriptive statistics as shown above on table 4, the time to maturities available are 2 years, 4years, 5years, 9years, 12years, 14years, 10years and 29 years with respective mean yield of 5.0506, 5.4797, 5.9325, 6.9495, 7.5734, 7.5595, 8.0615 and 7.9749, see table 5. The overall statistics experience a higher level of variance. These can be attributed to the numerous data and the monthly market behavior. The aggregate yield curve of as depicted on figure 5above shows that the yield curve is sloping upward giving an increase of time to maturity, but a decline was experienced on the twenty nine years time to maturity.

Table 6. First Quarter Descriptive Statistics 2019

Maturity	N	Min	Max	Mean	Std. Deviation	Variance
2 years	57	4.61	7.38	5.4007	.83809	.702
3 years	57	4.96	8.27	5.9854	1.02421	1.049
4 years	57	5.20	8.39	6.3392	.97386	.948
6 years	57	6.16	8.78	7.0964	.76344	.583
8 years	57	6.52	9.15	7.4145	.77638	.603
11 years	57	.26	9.07	7.5316	1.12999	1.277
12 years	57	7.24	9.10	7.9378	.47036	.221
13 years	57	7.23	9.19	7.9474	.50885	.259
19 years	57	7.63	9.55	8.2985	.55425	.307
28 years	38	7.76	8.32	8.0182	.13909	.019
30 years	38	8.12	8.80	8.4198	.18238	.033
Valid N (listwise)	38					

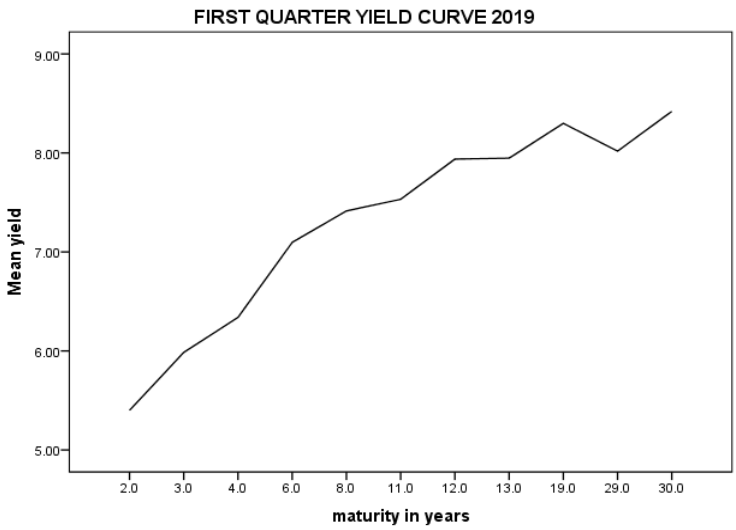

Figure 6: First Quarter Yield Curve 2019

From the above table 6of the first quarter comprising of January, February and march, the data obtained contained 11time to maturity which include two years, three years, four years, six years, eight years, eleven year, twelve years, thirteen years, nineteen years, twenty eight years and third years with a respective mean yield of 5.4007, 5.9854, 6.3392, 7.0964, 7.4145, 7.5316, 7.9378, 7.9474, 8.2985, 8.0182 and 8.4198.

From the figure 6above, it is observed that the yield curve is sloping upward with a little decline on twenty eight years. This means that lower maturities have lower percentage yield while the higher maturities have higher yield.

Table 7. Second Quarter Descriptive Statistics

Maturity	N	Min	Max	Mean	Std. Deviation	Variance
2 years	58	3.78	4.93	4.5728	.28082	.079
3 years	58	4.30	5.21	4.9696	.24269	.059
4 years	58	4.71	5.68	5.3845	.20950	.044
6 years	58	5.82	6.75	6.3276	.19132	.037
8 years	58	6.44	7.41	6.8594	.24547	.060
11 years	58	6.97	7.83	7.2950	.24243	.059
12 years	58	7.24	8.09	7.5698	.22751	.052
13 years	58	7.28	8.17	7.5870	.25028	.063
19 years	58	7.52	8.40	7.8822	.20709	.043
28 years	58	7.64	8.59	8.0392	.23651	.056
30 years	58	8.05	8.98	8.4438	.21949	.048
Valid N (listwise)	58					

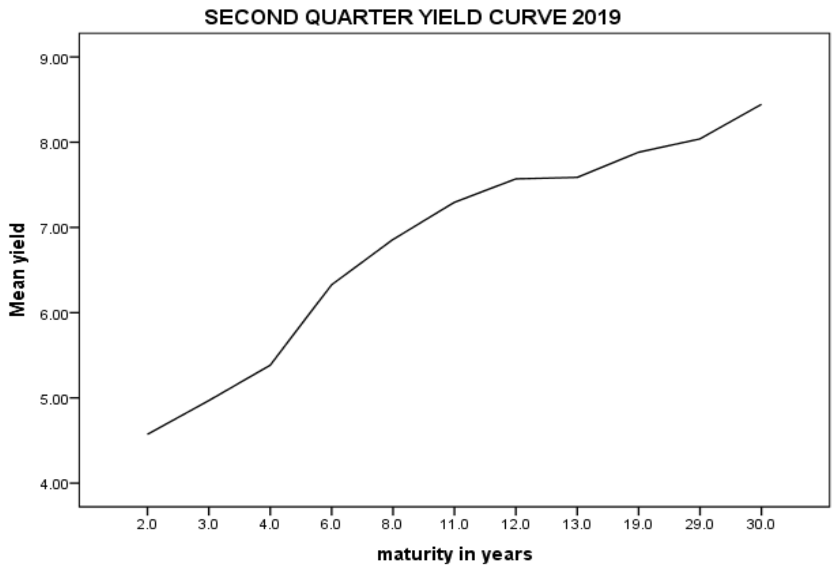

Figure 7: Second Quarter Yield Curve 2019

From the above table 7of the second quarter comprising of April, May, and June, the data obtained also contained11 time to maturity which include two years, three years, four years, six years, eight years, eleven year, twelve years, thirteen years, nineteen years, twenty eight years and third years with a respective mean yield of 4.5728, 4.9696, 5.3845, 6.3276, 6.8594, 7.2950, 7.5698, 7.5870, 7.8822, 8.0392 and 8.443.

From the figure7above, it is observed that the yield curve is sloping upward meaning that lower maturities have lower percentage yield while the higher maturities have higher yield.

Table 8. Third Quarter Descriptive Statistics

	N	Min	Max	Mean	Std. Deviation	Variance
2 years	64	3.65	4.35	3.9697	.15968	.025
3 years	64	4.00	4.49	4.2388	.10736	.012
4 years	64	4.35	4.99	4.7192	.15986	.026
6 years	64	5.33	5.91	5.6287	.12947	.017
8 years	64	5.88	6.73	6.3149	.18852	.036
11 years	64	6.50	7.31	6.8504	.19787	.039
12 years	64	6.88	7.61	7.2074	.18692	.035
13 years	64	6.91	7.73	7.2630	.21188	.045
19 years	64	.00	7.98	7.1676	1.61428	2.606
28 years	64	7.34	8.31	7.7320	.23540	.055
30 years	64	7.76	8.67	8.1380	.21627	.047
Valid N (listwise)	64					

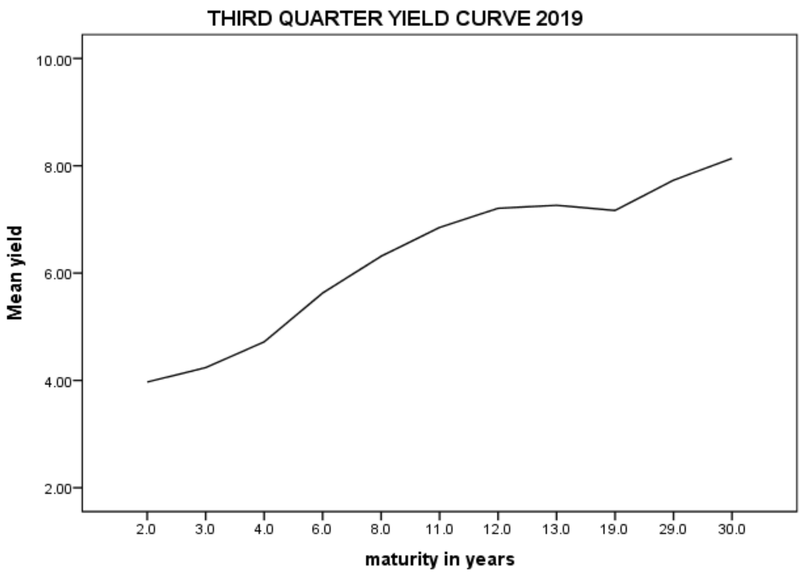

Figure 8: Third Quarter Yield Curve 2019

From the above table 8of the third quarter comprising of July, August and September, the data obtained also contained11 time to maturity which include two years, three years, four years, six years, eight years, eleven year, twelve years, thirteen years, nineteen years, twenty eight years and third years with a respective mean yield 3.9697, 4.2388, 4.7192, 5.6287, 6.3149, 6.8504, 7.2074, 7.2630, 7.1676, 7.7320 and 8.1380 From the figure 8above, it is observed that the yield curve is sloping upward meaning that lower maturities have lower percentage yield while the higher maturities have higher yield.

Table 9. Fourth Quarter Descriptive Statistics

Maturity	N	Min	Max	Mean	Std. Deviation	Variance
2 years	61	3.07	4.10	3.5513	.31248	.098
3 years	61	3.77	4.76	4.1457	.23782	.057
4 years	61	4.25	5.07	4.6639	.24672	.061
6 years	61	5.50	6.00	5.7485	.16222	.026
8 years	61	6.13	7.89	6.4207	.25073	.063
11 years	61	6.85	7.36	7.0478	.13828	.019
12 years	61	7.19	7.64	7.3826	.10533	.011
13 years	61	7.24	7.83	7.4867	.14124	.020
19 years	61	.00	8.07	7.3617	1.69339	2.868
28 years	61	7.66	8.30	7.9281	.14136	.020
30 years	61	8.05	8.62	8.3005	.13560	.018
Valid N (listwise)	61					

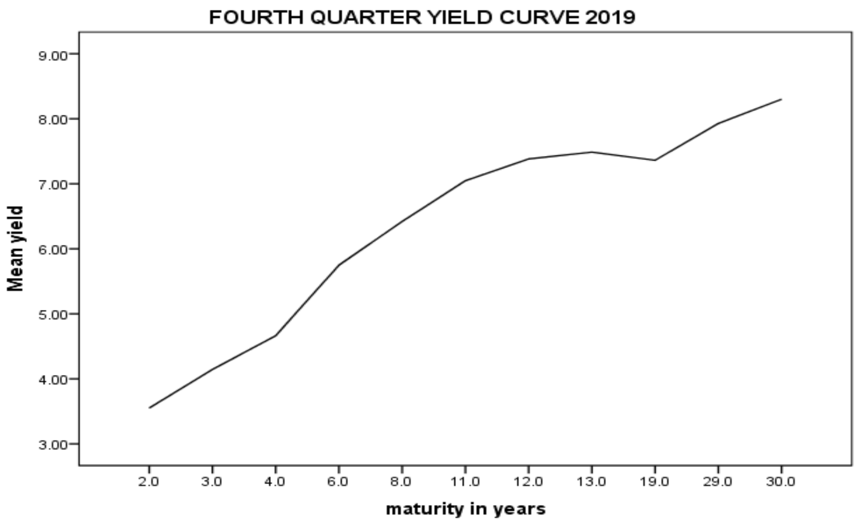

Figure 9: Fourth Quarter Yield Curve 2019

From the above table 9of the second quarter comprising of October, November and December, the data obtained also contained11 time to maturity which include two years, three years, four years, six years, eight years, eleven year, twelve years, thirteen years, nineteen years, twenty eight years and third years with a respective mean yield 3.5513, 4.1457, 4.6639, 5.7485, 6.4207, 7.0478, 7.3826, 7.4867, 7.3617, 7.9281 and 8.3005. From the figure 4above, it is observed that the yield curve is sloping upward with a decline on nineteen years maturity. This means that lower maturities have lower percentage yield while the higher maturities have higher yield.

Table 10. Overall Descriptive Statistics 2019

Maturity	N	Min	Max	Mean	Std. Deviation	Variance
2 years	240	3.07	6.13	4.2721	.64950	.422
3 years	240	3.77	6.60	4.6943	.61453	.378
4 years	240	4.25	7.38	5.1620	.61157	.374
6 years	240	5.33	8.27	6.1185	.55566	.309
8 years	240	5.88	8.39	6.6706	.44338	.197
11 years	240	6.50	8.78	7.1809	.38830	.151
12 years	240	6.88	9.15	7.5085	.37793	.143
13 years	240	.26	9.07	7.5123	.58821	.346
19 years	240	.00	9.10	7.6163	1.26190	1.592
28 years	240	7.34	9.19	7.9684	.29682	.088
30 years	240	7.76	9.55	8.3684	.29902	.089
Valid N (listwise)	240					

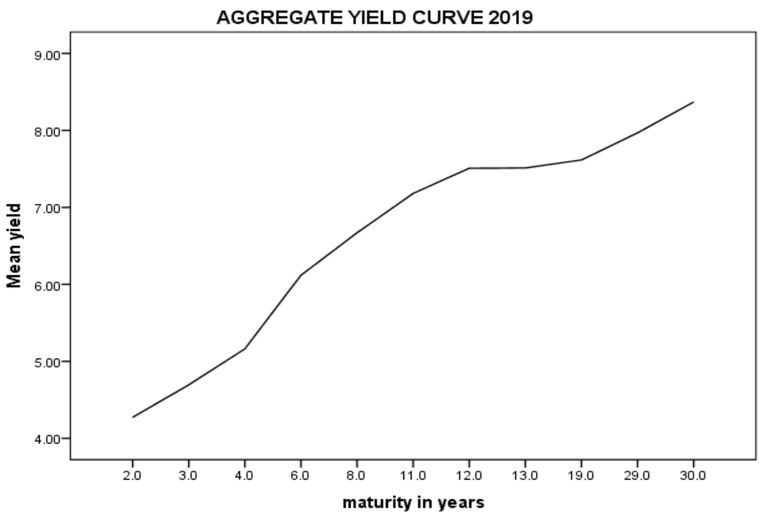

Figure 10: Aggregate Yield Curve 2019

The above table 10depicted the overall descriptive statistics which will be used for further analysis of the model. The maturities in the is 11which include two years, three years, four years, six years, eight years, eleven year, twelve years, thirteen years, nineteen years, twenty eight years and third years with a respective mean yield of 4.2721, 4.6943, 5.1620, 6.1185, 6.6706, 7.1809, 7.5085, 7.5123, 7.6163, 7.9684 and 8.3684The above figure 5showed the mean yield curve for the year 2019of the Nigerian Eurobond. The yield curve is upward sloping indicating that the higher the time to maturity, the higher the yield. In other words, time to maturity is directly proportional to its yield.

2.5 Research Questions

How Do We Predict the In-Sample Field of τ?

As already established that the Nelson-Siegel model can be employed to estimate in-sample of τ (τ as time to maturity) given an observed yield of bond, the model parameters were calculated using the ordinary least square method given the value $\lambda = 0.037782$we set τ in months. The result is presented in table 1 below.

Table 11. Coefficients 2018

Model parameters		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
	β_0	8.864	.192		46.206	.000
	β_1	-4.369	.605	-.748	-7.223	.001
	β_2	-4.004	1.483	-.280	-2.700	.043

Authors' computation

Table 12 Coefficients 2019

Model parameters		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
	β_0	8.812	.145		60.574	.000
	β_1	-6.290	.492	-.891	-12.774	.000
	β_2	-2.021	1.180	-.119	-1.713	.125

Authors' computation

In order to obtain the discount function$D(\xi) = e^{-\xi\Upsilon(\xi)}$, we substitute the parameters computed parameters from the above table 11 and table 12 into the model for 2018 and 2019 respectively as follows.

$$D(\tau) = e^{-\tau\left\{8.864-4.369\left(\frac{1-e^{-0.03778438\tau}}{0.03778238\tau}\right)-4.004\left(\frac{1-e^{-0.03778238\tau}}{0.03778238\tau}-e^{-0.03778238\tau}\right)\right\}} \quad (47)$$

In practice, the Nelson-Siegel model is often applied in the analysis and hedging of the interest rate risk of insurance portfolios with defined flows.

where $y_t(\tau) = 8.864 - 4.369\left(\frac{1-e^{-0.03778438\tau}}{0.03778238\tau}\right) - 4.004\left(\frac{1-e^{-0.03778238\tau}}{0.03778238\tau} - e^{-0.03778238\tau}\right)$

$0 \leq \tau \leq 348$ (48)

and $D(\tau) = e^{-\tau\left\{8.812-6.290\left(\frac{1-e^{-0.03778438\tau}}{0.03778238\tau}\right)-2.021\left(\frac{1-e^{-0.03778238\tau}}{0.03778238\tau}-e^{-0.03778238\tau}\right)\right\}}$ (49)

$$y_t(\tau) = 8.812 - 6.290\left(\frac{1-e^{-0.03778438\tau}}{0.03778238\tau}\right) - 2.021\left(\frac{1-e^{-0.03778238\tau}}{0.03778238\tau} - e^{-0.03778238\tau}\right)$$

$0 \leq \tau \leq 360$ (50)

at any given value provided it does not exceeds 348 months for 2018 and 360 months for 2019, yield can be estimated using the above models.

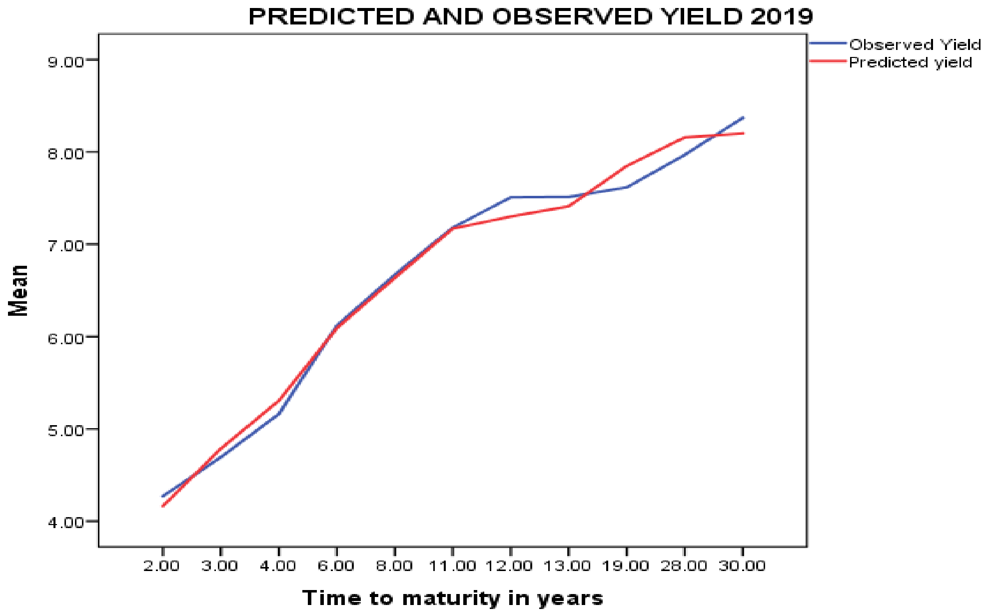

Figure 11: Predicted and Observed Yield 2019

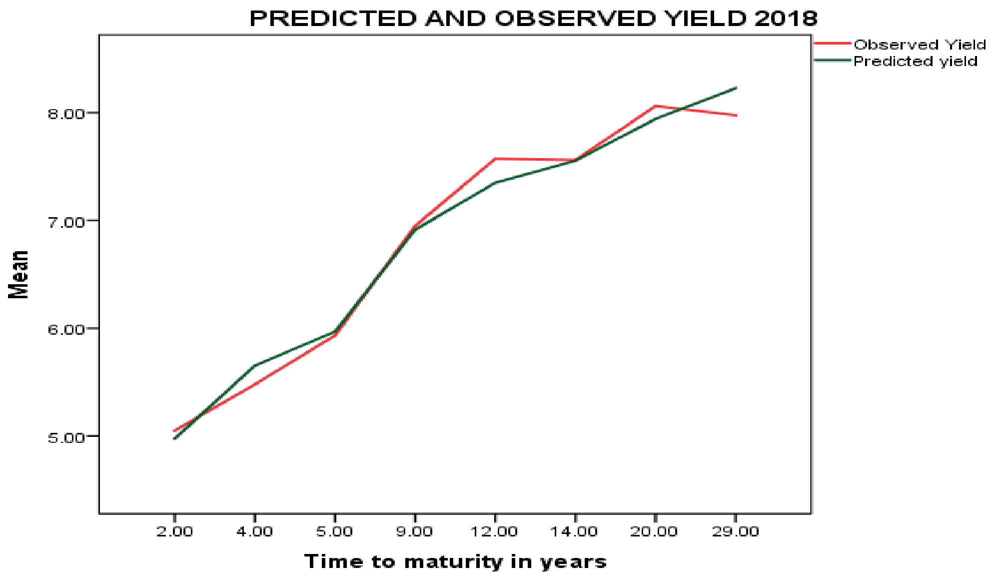

Figure 12: Predicted and Observed Yield 2018

***How Do We Use the Parameters to Determine Long-Term Yield and Short-term Yield*?**

The computed parameters can be applied to obtain the short term yield and the long-term yield. As defined by (Diebold, & Rudenbusch, 2013), the parameter$\alpha_0$is defined as long-term yield and $\alpha_0 + \alpha_1$ as short-term yield. Following the definition, the long-term yield for2018and2019are 8.864and 8.812respectively which is so close to the29years and30years yield of 7.9749and 8.3684for the two years. We can conclude that from the observed data, the long-term yield isα_0. The value of the defined short-term yield is4.495and2.522and the observed short-term is the two years maturity with the mean yield of 5.0506 and 4.2421with the difference of 0.5556 and 1.7201 for 2018 and 2019 respectively. The difference between the observed and the defined yield is significantly large, we can be concluded that the defined short-term cannot be referred to as the short-term yield.

How Does the Model Fit into the Observed Data?

The measure goodness of fit is obtained when ordinary least square method is applied on data by R^2 adjusted. The model measure of fit analysis is depicted in table 13 below

Table 13. Model summary 2018

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.991[a]	.983	.976	.18252

Table 14. Model summary 2019

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.994[a]	.989	.986	.16373

The adjusted$R$ square is the coefficient of multiple determinations which is the variance percentage in the dependent variable as explained by the independent variable. From the table13 and 14for 2018and 2019, theR^2adjusted is 0.976and 0.986. This indicates that 97.6% of 2018and 98.6% of 2019 observed data can be explained by the Nelson-Siegel model using the estimated parameters. We can conclude that the Nelson-Siegel model in this study fits very well. The adjusted $R^2 = \frac{unexplained\ variation}{total\ variation}$ measures the bond's price movement which can be explained by the oscillations in a benchmark index. From the tables13 and 14; 97.6%of 2018and 98.6% of 2019high vale indicates that the bond's performance oscillates in line with index will most likely offer higher risk adjusted returns.

3. *Discussion of Findings*

From the descriptive analysis of the Nigerian Eurobond for both 2018 and 2019, we found that the slope of the yields has an upward movement. These indicate that as time to maturity increases, the corresponding yield increases. Consequently, we can conclude that time to maturity and its yield has a direct proportionality relationship. Also, from the fact that as time to maturity increases its yield increase reveals the premise that the higher the risk, the higher the expected return. This is to compensate investors who part their money for long time. In the analysis of the Nelson-Siegel model we found that the model fits in well with the observed data. This was revealed by the adjusted Rsquare for 2018 and 2019 of 0.976and0.986. This indicates that 97.6%of 2018 and 98.6%of the observed data can be explained by the Nelson-Siegel model of data using the estimated parameters for the two years.

Also, using the interpretation in Diebold & Rudenbusch (2013), that the parameter α_0 is as long-term yield and $\alpha_0 + \alpha_1$ as short-term yield. We found out that despite the difference indicated by the value the long-term yield and α_0, we can conclude that from the observed data, the long-term yield is α_0. However, in the case of $\alpha_0 + \alpha_1$ as short-term yield, the difference showed a high disparity. The predicted yields forβ_1, β_2, β_3 are shown in tables 15, table 16, table 17, table 18, table 19and table 20while the behavior of the trajectories are depicted in figures 13, figure 14, figure15, figure16, figure17and figure18. Observe that the steepness of the yield curve indicates a condition where the shape of the bond interest rate curve varies from a concave curve to a shape approaching a straight line and the steepness of the curve is formed when investors ask for big risk premium to lend at higher term maturities to hedge against inflationary risk that they perceive larger than the current position. The steepness of the yield curve is either connected with expectations of an increase in forward inflation or a decline in the fiscal position. Given the estimated parametric model, we predict the yields would increase such that the yield curve will be upward sloping. A cogent reason behind this prediction is the comparatively unfavourable economic condition in Nigeria associated with liquidity problems suspected to impact more on the yield curve in the future either by staying above if the condition becomes worst or by staying below if the economic condition is improved in the future.

Table 15. Predicted Yield with Different Values of β_0 2018

Maturity	$\boldsymbol{\beta_0}$= 8.864	$\boldsymbol{\beta_0}$=9.864	$\boldsymbol{\beta_0}$=10.864	$\boldsymbol{\beta_0}$=11.864
2 years	4.975929	5.975929	6.975929	7.975929
4 years	5.652926	6.652926	7.652926	8.652926
5 years	*5.96815*	6.968147	7.968147	8.968147
9 years	6.914368	7.914368	8.914368	9.914368
12years	7.349057	8.349057	9.349057	10.34906
14 years	7.554195	8.554195	9.554195	10.55419
20 years	7.941179	8.941179	9.941179	10.94118
29 years	8.227189	9.227189	10.22719	11.22719

Figure 13: Predicted Yield with Different Values of β_0 2018

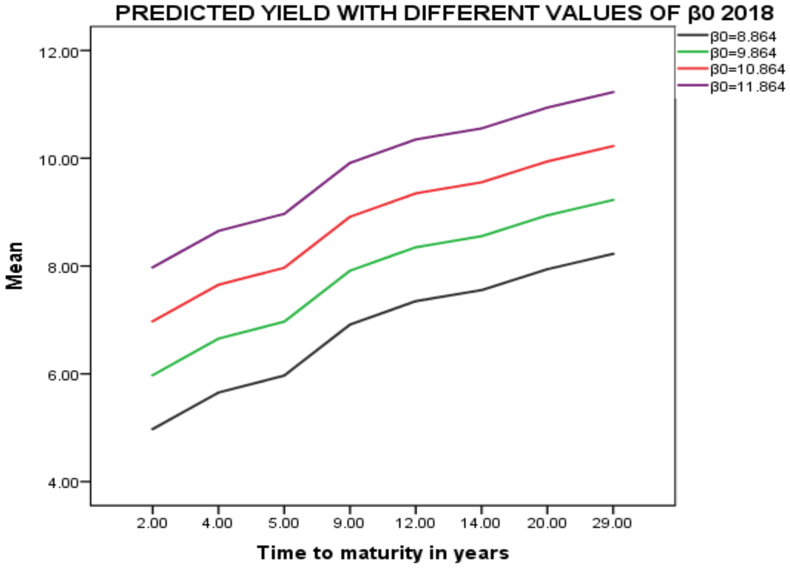

Table 16. Predicted Yield with Different Values of β_1 2018

Maturity	$\boldsymbol{\beta_1}$=-4.369	$\boldsymbol{\beta_1}$=-3.369	$\boldsymbol{\beta_1}$= -2.369	$\boldsymbol{\beta_1}$=-1.369
2 years	4.975929	5.633399	6.290868	6.948338
4 years	5.652926	6.114414	6.575901	7.037388
5 years	5.968147	6.36356	6.758972	7.154385
9 years	6.914368	7.155297	7.396226	7.637155
12years	7.349057	7.532063	7.715068	7.898074
14 years	7.554195	7.711464	7.868734	8.026003
20 years	7.941179	8.051448	8.161717	8.271986
29 years	8.227189	8.303245	8.379301	8.455358

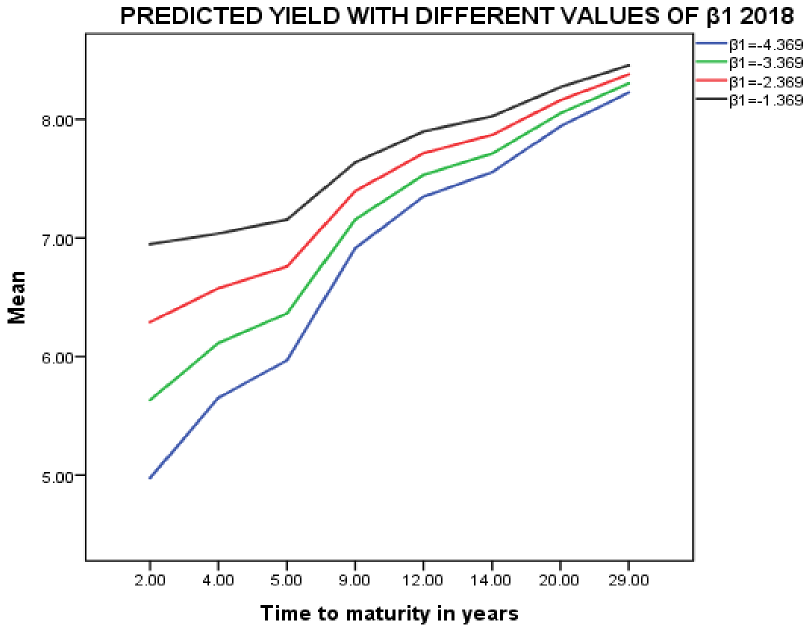

Figure 14: Predicted Yield with Different Values of β_1 2018

Table 17. Predicted Yield with Different Values of β_2 2018

Maturity	$\boldsymbol{\beta_2}$= -4.004	$\boldsymbol{\beta_2}$= -3.004	$\boldsymbol{\beta_2}$= -2.004	$\boldsymbol{\beta_2}$= -1.004
2 years	4.975929	5.229571	5.48321	5.736857
4 years	5.652926	5.951337	6.24975	6.548158
5 years	5.968147	6.259929	6.55171	6.843493
9 years	6.914368	7.138397	7.362427	7.586456
12years	7.349057	7.527726	7.706395	7.885064
14 years	7.554195	7.709713	7.865231	8.020749
20 years	7.941179	8.051333	8.161486	8.27164
29 years	8.227189	8.303243	8.379297	8.455352

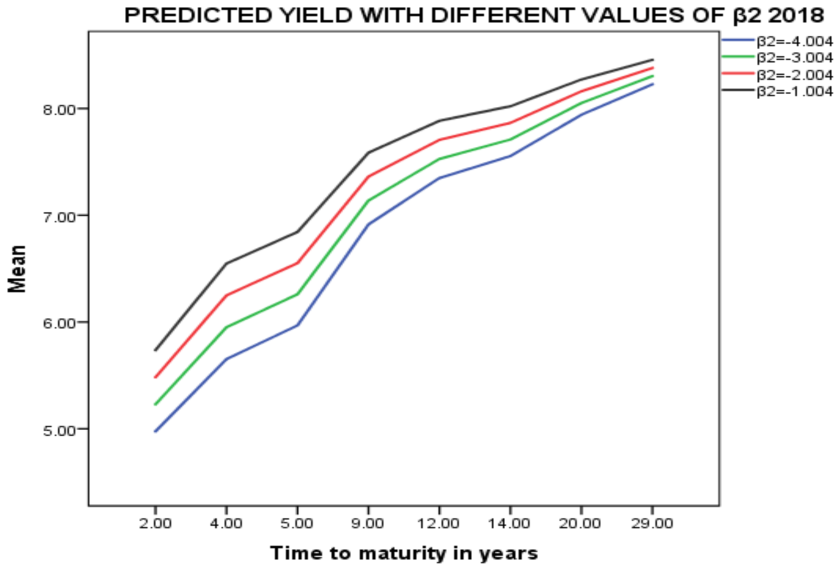

Figure 15: Predicted Yield with Different Values of β_2 2018

Table 18. Predicted Yield with Different Values of β_0 2019

Maturity	$\boldsymbol{\beta_0}$= 8.812	$\boldsymbol{\beta_0}$=9.812	$\boldsymbol{\beta_0}$=10.812	$\boldsymbol{\beta_0}$=11.812
2 years	4.163902	5.163902	6.163902	7.163902
3 years	4.788334	5.788334	6.788334	7.788334
4 years	5.306158	6.30616	7.306158	8.306158
6 years	6.09112	7.091117	8.091117	9.091117
8 years	6.635303	7.635303	8.635303	9.635303
11 years	7.170707	8.170707	9.170707	10.17071
12 years	7.299803	8.299803	9.299803	10.2998
13 years	7.411376	8.411376	9.411376	10.41138
19 years	7.84775	8.84775	9.84775	10.84775
28 years	8.1799	9.1799	10.1799	11.1799
30 years	8.200969	9.20097	10.201	11.201

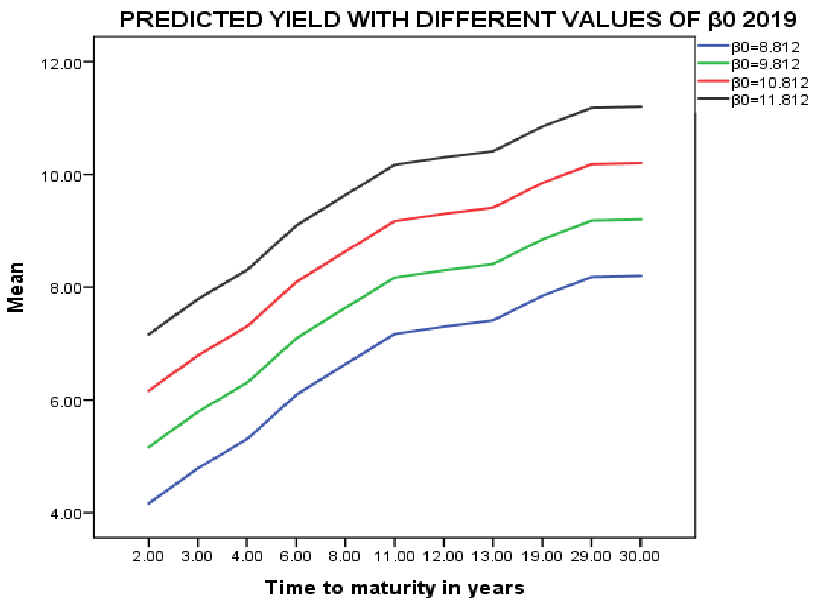

Figure 16: Predicted Yield with Different Values of β_0 2019

Table 19. Predicted Yield with Different Values of β_1 2019

Maturity	$\boldsymbol{\beta_1}$= -6.290	$\boldsymbol{\beta_1}$= -5 .290	$\boldsymbol{\beta_1}$= -4.290	$\boldsymbol{\beta_1}$= -3 .290
2 years	4.163902	4.821372	5.478842	6.136312
3 years	4.788334	5.334875	5.881415	6.427956
4 years	*5.30616*	5.767645	6.229132	6.690619
6 years	6.091117	6.434514	6.777912	7.121309
8 years	6.635303	6.903676	7.172048	7.44042
11 years	7.170707	7.369851	7.568995	7.768139
12 years	7.299803	7.482809	7.665815	7.848821
13 years	7.411376	7.580573	7.749769	7.918966
19 years	7.84775	7.963815	8.07988	8.195945
28 years	8.1799	8.255957	8.332013	8.408069
30 years	*8.20097*	8.27449	8.348011	8.421532

Source: Authors' computation

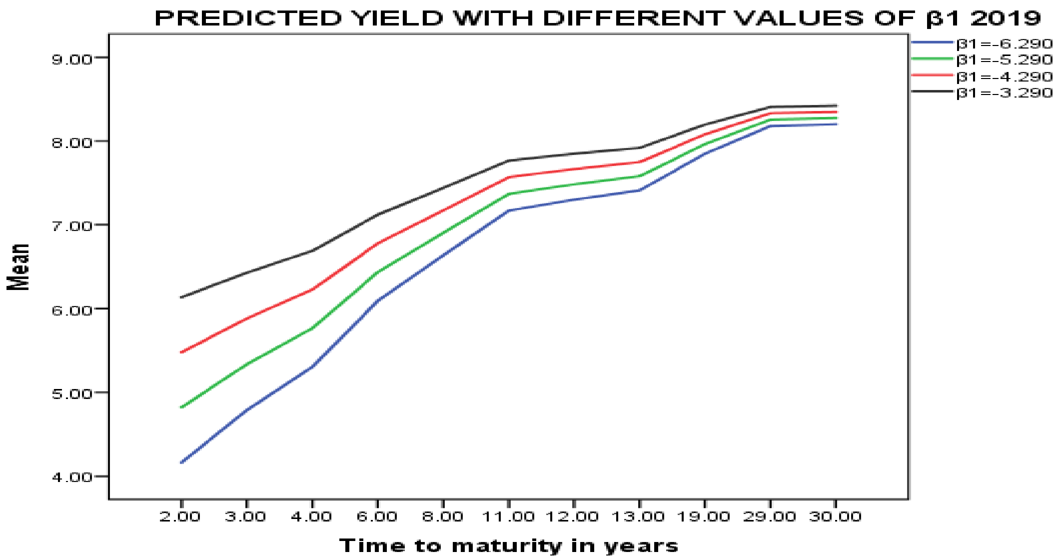

Figure 17: Predicted Yield with Different Values of β_1 2019

Table 20. Predicted Yield with Different Values of β_2 2019

Maturity	$\boldsymbol{\beta_2}$=-2.021	$\boldsymbol{\beta_2}$=-1 .021	$\boldsymbol{\beta_2}$=-0 .021	$\boldsymbol{\beta_2}$= 0.979
2 years	4.163902	4.417545	4.67119	4.92483
3 years	4.788334	5.078253	5.36817	5.65809
4 years	*5.30616*	5.604569	5.90298	6.20139
6 years	6.091117	6.368659	6.646202	6.923745
8 years	6.635303	6.877082	7.11886	7.360639
11 years	7.170707	7.363027	7.555346	7.747665
12 years	7.299803	7.478472	7.657141	7.83581
13 years	7.411376	7.577817	7.744257	7.910698
19 years	7.84775	7.963634	8.079517	8.195401
28 years	8.1799	8.255955	8.332009	8.408063
30 years	8.20097	8.274488	8.348008	8.421528

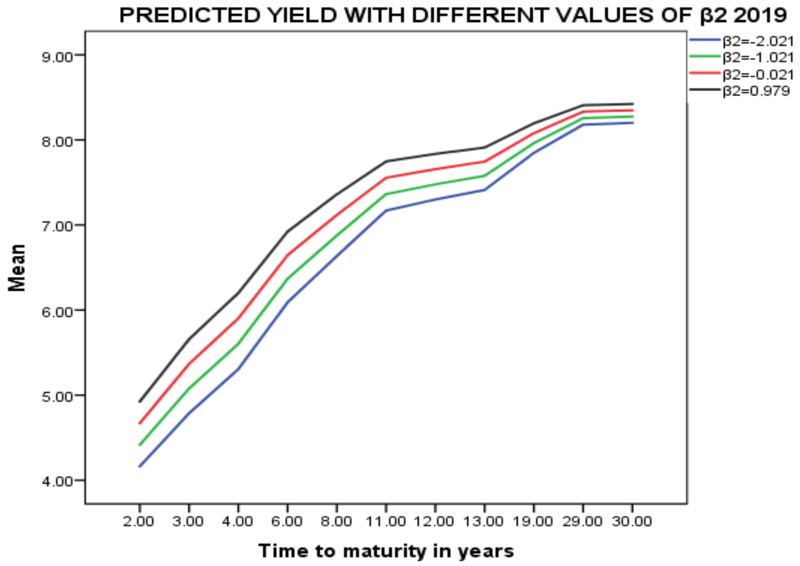

Figure 18: Predicted Yield with Different Values of β_2 2019

We compare our results with standard results in Muvingi and Kiwinjo (2014) who similarly conducted $t - tests$with R^2while estimating the term structure of interest rate under Nelson-Siegel framework. The authors obtained a very low R^2. We can reasonably conclude that our relatively high R^2result indicates better performance.

4. Conclusion

In practice, the Nelson-Siegel model is often applied in the analysis and hedging of the interest rate risk of insurance portfolios with defined flows. The model has the advantage of being parsimonious and its parameters has great economic significance. Using the Nelson-Siegel model, a normal yield curve trajectory was produced showing the statistical evidence that the bigger the maturity is, the bigger the interest rate becomes. The main rationale for the upward sloping of the yield curve noticed in our results could be associated with lack of market liquidity and visibility of the future economic conditions in Nigeria as a whole. Currently, Nigeria has been raising its level of nominal debts categorized as internal and external debts. The critical problem of Nigerian government is the increasing level of external debt reinforced by the internal debts. To provide solution to this disequilibrium between the two categories of debts, the Nigerian government should be funded by the banking sector and the funding must be set up through the bond market. Furthermore, the yield curve is generated through the interest rate of government bonds by maturities with motions which show the interest rates oscillations described by Nigerian monetary policies which sets the targets for long-term and short-term rates. Consequently, this yield curve control policy critically impacts on the fluctuation behaviour of the yield curve since it sets a target for long term and short term interest rates. When the economy seems inactive, the central bank stimulates the economy by reducing the policy rate to create an investment environment where investors and government could raise funds. In Nigeria, the shape of the yield curve has changed in recent years as a result of the changes in monetary policies. Consequently, it is pertinent to employ a functional model which can flexibly respond to the different shapes of the yield curve. This necessity lead to the application of the Nelson-

Siegel's with exponential components to model the Nigerian discount function through its yield curve as a parsimonious three parameter continuous function. As a direction for future research, applied stochastic methods may be used to obtain the discount function and the guaranteed minimum maturity benefit.

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